SHARP ESTIMATE OF PROPAGATION FOR THE FISHER-KPP POROUS MEDIUM EQUATION

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Abstract: We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and nonnegative, compactly supported initial function in $L^\infty(R^N) \setminus \{0\}$:

$$(*) \quad u_t = \Delta u^m + u - u^2 \quad \text{in } Q := R^N \times R_+, \qquad u(\cdot, 0) = u_0 \quad \text{in } R^N,$$

It is well known that the spatial support of the solution $u(\cdot, t)$ to this problem remains bounded for all time $t > 0$. In spatial dimension one it is known that there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution U_{c_*} with a finite front, and this traveling wave solution is asymptotically stable in the sense that if the initial function $u_0 \in L^\infty(R)$ satisfies $\liminf_{x \to -\infty} u_0(x) > 0$ and $u_0(x) = 0$ for all large x, then $\lim_{t \to \infty} \{\sup_{x \in R} |u(x,t) - U_{c_*}(x - c_* t - x_0)|\} = 0$ for some $x_0 \in R$. In higher dimensions we show that U_{c_*} is still attractive, albeit that a logarithmic shifting occurs. More precisely, if the initial function in $(*)$ is additionally assumed to be radially symmetric, then there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t \to \infty} \left\{ \sup_{x \in R^N} |u(x,t) - U_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in R$ (depending on u_0). If the initial function is not radially symmetric, then there exist $r_1, r_2 \in R$ such that the boundary of the spatial support of the solution $u(\cdot, t)$ for all large time t is contained in the spherical shell $\{x \in R^N : r_1 \leq |x| - c_* t + (N-1)c^* \log t \leq r_2\}$, and for any $c \in (0, c^*)$, $\lim_{t \to \infty} u(x,t) = 1$ uniformly in $\{|x| \leq c_* t - (N-1)c \log t\}$.

This is based on joint work with Fernando Quiros (Univ Autonoma Madrid) and Maolin Zhou (Nankai Univ).